Wednesday November 2, 2022

Reminders

- MLM on Related Rates due Thursday

2 of them!

- Written HW 4 due Wednesday
- Office hours today 1-2³⁰ in Locy 203

Recap

Point-slope form of a line

$$y - y_0 = m(x - x_0)$$

3.9 Linearization

"A linearization is a tangent line with a job"

Ex 1. Find the linear approximation to $f(x) = \frac{1}{(1+2x)^4}$ at $x=0$ and use it to estimate $f(0.1)$.

$f'(x) = -4(1+2x)^{-5} \cdot 2$

$f'(0) = -4(1+0)^{-5} \cdot 2$

$= -8$

slope

$f(0) = 1$

point

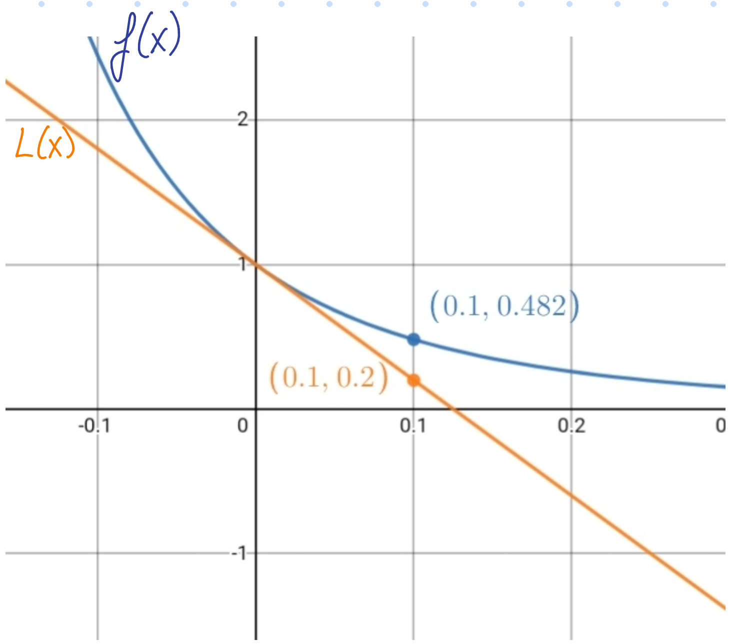
Tangent line: $y - 1 = -8(x-0)$

Linearization: $L(x) = -8x + 1$

$f(0.1) \approx L(0.1) = -8(0.1) + 1$

$= -.8 + 1$

$= \boxed{0.2}$



Ex 2. Use a linear approximation to estimate $(8.06)^{2/3}$ by hand.

$8.06^{2/3} = \left(\sqrt[3]{8}\right)^2 \approx \left(\sqrt[3]{8}\right)^2$ Actually computable by hand. It's 4

Let $f(x) = x^{2/3}$ and build linear approximation at $x = 8$

$f(8) = 4$ $\quad f'(x) = \frac{2}{3} x^{-1/3}$

point $\quad f'(8) = \frac{2}{3}(8)^{-1/3}$

$= \frac{1}{3}$ slope

Tangent line: $y - 4 = \frac{1}{3}(x - 8)$

Linearization: $L(x) = \frac{1}{3}(x - 8) + 4$

$$\begin{aligned}
(8.06)^{2/3} &= f(8.06) \\
&\approx L(8.06) \\
&= \tfrac{1}{3}(8.06 - 8) + 4 \\
&= \tfrac{1}{3}(.06) + 4 \\
&= \boxed{4.02}
\end{aligned}$$

Ex 3. Atmospheric pressure P decreases as altitude h increases. At a temperature of 15° C, the pressure is 101.3 kPa at sea level, 87.1 kPa at an altitude of 1 km, and 74.9 kPa at an altitude of 2 km. Use a linear approximation to estimate the atmospheric pressure at an altitude of 3 km.

$$m = \frac{74.9 - 87.1}{2-1} = \frac{-12.2}{2} = -6.1$$ ← slope

(2, 74.9) ← point

$$y - 74.9 = -6.1(x-2)$$

$$L(x) = -6.1(x-2) + 74.9$$

$$L(3) = -6.1(3-2) + 74.9$$

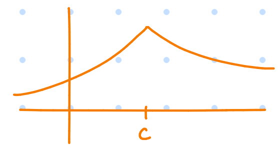
$$= \boxed{68.8 \text{ kPa}}$$

Ex 4. Ponder...

(a) Does a function always have a linearization at a given point $x=c$?

No, a linearization is not possible at points where f is not differentiable

ex.)



No tangent line possible at $x=c$

(b) If $f(x)$ has a linearization at $x=c$, is the linearization unique?

Yes.

(c) When is a function $f(x)$ equal to its linearization?

When $f(x)$ is a line.

(d) If the linearization to $f(x)$ at $x=c$ is $L(x)$, is there a max or min on the error of using $L(x)$?

The error between $L(x)$ and $f(x)$ can be zero if $f(x)$ is a line.
There is no max cap on the error.

(e) What are some ideas for getting an approximation to $f(x)$ that's better than the linearization?

Higher order approximations

Newton's method